

Problem T-1

Determine all functions $f: \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(x)f(y+z) \geq x(f(y) + f(z))$$

for all $x, y, z \in \mathbb{R}$.

Problem T-2

Determine all positive rational numbers q for which there exist positive integers a , b , c , and n satisfying the following two equations:

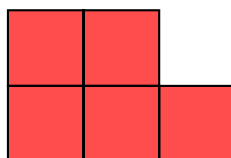
$$a + b + c = qn,$$

$$ab + c = n^2.$$

Problem T-3

Consider an infinite square grid consisting of white cells. Let n be a positive integer. Kekec coloured $4n + 1$ of the cells red such that both the set R of red cells and the set W of white cells are connected.

The resultant red shape is a k -gon. For example, the following 5 red cells form a 6-gon:



Determine the largest possible value of k in terms of n .

Remark: A set S of cells is said to be *connected*, if for any two cells $a, b \in S$, there exists an integer $m \geq 1$ and cells $c_1, c_2, \dots, c_m \in S$ where $c_1 = a$, $c_m = b$, and cells c_i, c_{i+1} for $1 \leq i \leq m-1$ share an edge.

Problem T-4

Let n be a positive integer. A treasure chest has $2n - 1$ locks on it labeled from 1 to $2n - 1$. There are $n \cdot (2n - 1)$ keys, each opening exactly one lock. For each $1 \leq i \leq 2n - 1$, there are i keys that open the i^{th} lock. All these keys are distributed to $2n - 1$ people so that each person owns n keys (each person can own multiple keys opening the same lock).

Determine the smallest positive integer k , such that no matter how the keys are distributed to the $2n - 1$ people, there exists a group of k people that can open all the locks together.

Problem T–5

Let $ABCD$ be an isosceles trapezoid where $AD \parallel BC$ and $AD < BC$. Let O be the circumcenter of triangle ABC . Let X be the point such that BX is a diameter of the circumcircle of triangle ABO , and let Y be the point such that CY is a diameter of the circumcircle of triangle ACO .

Show that the points A , D , X , and Y lie on a circle.

Problem T–6

Let ABC be an acute triangle. Let I_a be its A -excenter. Points D and E lie on the lines I_aB and I_aC , respectively, and satisfy $DA = DB$ and $EA = EC$. The circumcircle of I_aDE intersects line BC at distinct points F and G . Prove that the circumcircles of ABC and AFG are tangent.

Remark: The A -excenter of the triangle ABC is the center of the circle outside the triangle which is tangent to the lines AB , AC , and the line segment BC .

Problem T–7

Let α be a positive rational number. In the beginning, the number 2026 is written on the blackboard. In each step, if x is the number currently written on the blackboard, we may replace it with one of the following numbers:

- i) $x + \alpha$,
- ii) $x - \alpha$,
- iii) $\frac{1}{x}$ if $x \neq 0$.

We call a rational number y *reachable* if it can appear on the blackboard after a finite number of steps.

Determine all positive rational numbers α for which *all* rational numbers are reachable.

Problem T–8

Determine all positive integers n with no prime divisor greater than 2026 such that for every choice of two *odd* and *coprime* positive divisors of n , their arithmetic mean is also a divisor of n .